PX 2018: Graph Drawing

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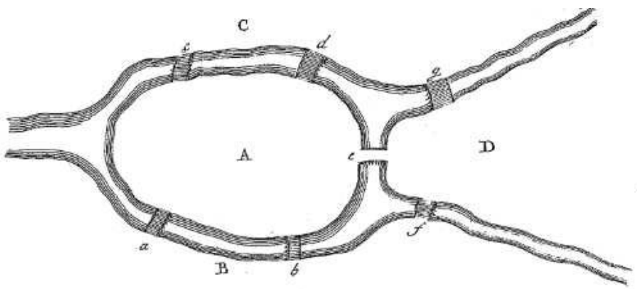
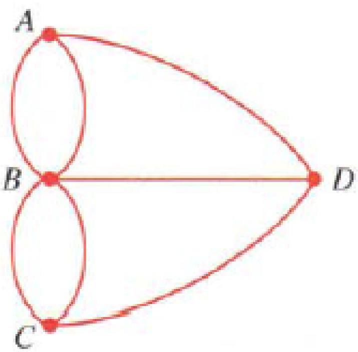
print

Motivation

- Traditional and powerful tools that visually represent sets of data and the relations among them
- Concept of graphs can be traced back to Ancient Egypt - Game Morris
- First scientific purposes 1736: Euler published his famous Königsberg paper ("drawing")
- Very first abstract graph drawing appeared in Ball's book on mathematical recreations 1892





sources:
http://stewartmath.com/dp_fops_samples/fops6.html
https://www.researchgate.net/figure/The-Seven-Bridges-of-Koenigsberg-problem-a-Eulers-drawing-31-b-Balls-abstract_fig1_325794369
http://www.instructables.com/id/Nine-Mens-Morris/

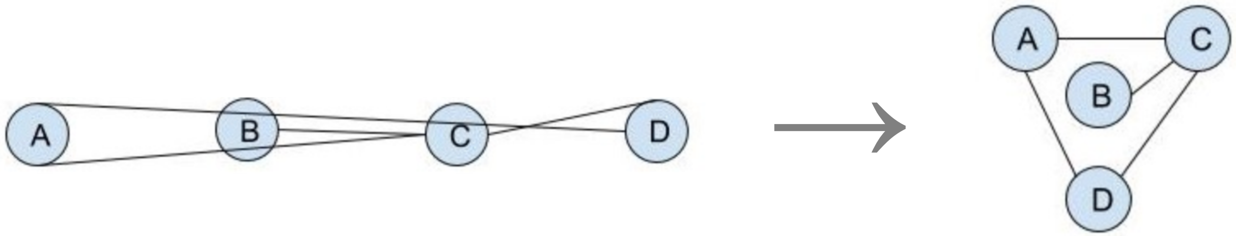
Key Challenges

- **Algorithm complexity:** graph size is crucial to algorithms
- **Display clutter:** when size of data grows, the corresponding graph becomes cluttered and visually confusing
- **Navigation:** navigating large information spaces on small displays
- → **Readability**

Key Requirements

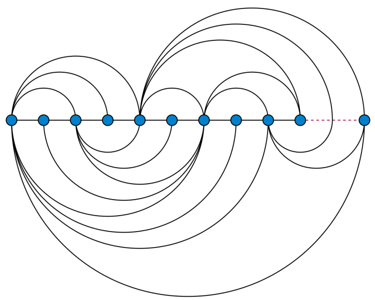
Improving readability through:

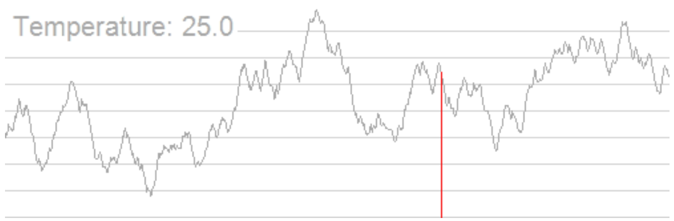
- Low number of crossing
- Small bounding box of graph
- Short edges



Layout Methods

- Arc Layout: vertices of a graph are placed along a line & minimizes the number of crossings.
- Circle Layout: places the vertices of a graph on a circle (network topologies)
- **Force-directed Layout**
- **Energy-minimizing simulations (Simulated Annealing)**





source
commons.wikimedia.org

Force-directed Graphs

General

Position the nodes of a graph in two-dimensional so that:

- all the edges are of more or less equal length
- few crossing edges
- by assigning forces among the set of edges and the set of nodes and using these forces to simulate the motion of the edges and nodes

First Pioneers:

- Eades 1984: Combination of attractive forces on adjacent vertices, and repulsive forces on all vertices
- Fruchterman & Reingold 1991

Algorithm

Dwyer's Implementation:

- Much faster and scalable to much larger force-directed graphs: O(nlogn+m+c)
- Providing users with interactive control over the layout
- Allowing users to achieve layout customized for their specific application or diagram.
- Implements three primary forces upon the nodes

source:
http://vis.stanford.edu/files/2011-D3-InfoVis.pdf
http://users.monash.edu/~tdwyer/Dwyer2009FastConstraints.pdf

Demo

Reset Controls | Medium Graph ▼

Alpha Simulation Activity

Center Shifts the view, so the graph is centered at this location.

X 0.5 Y 0.5

☑ **Charge** Attracts (+) or repels (-) nodes to/from each other.

Strength -30 Distance Minimum 1 Distance Maximum 2000

☑ **Collide** Prevents nodes from overlapping

Strength 0.7 Radius 5 Iterations 1

☐ **ForceX** Acts like gravity. Pulls all points towards an X location.

Strength 0.1

Evaluation

Advantages:

- Good quality
- Readability
- Interactivity
- Simplicity
- Bounding Boxed

Disadvantages:

- Can lead to jittering
- Possible high running time
- Not very stable

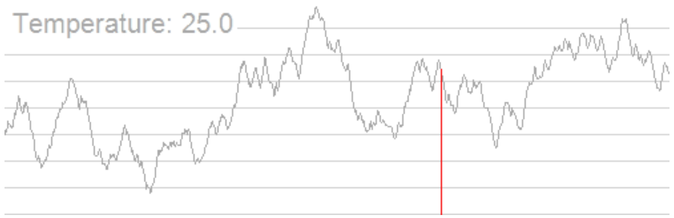
Simulated Annealing

Simulated Annealing

Source: homesteading.com

General

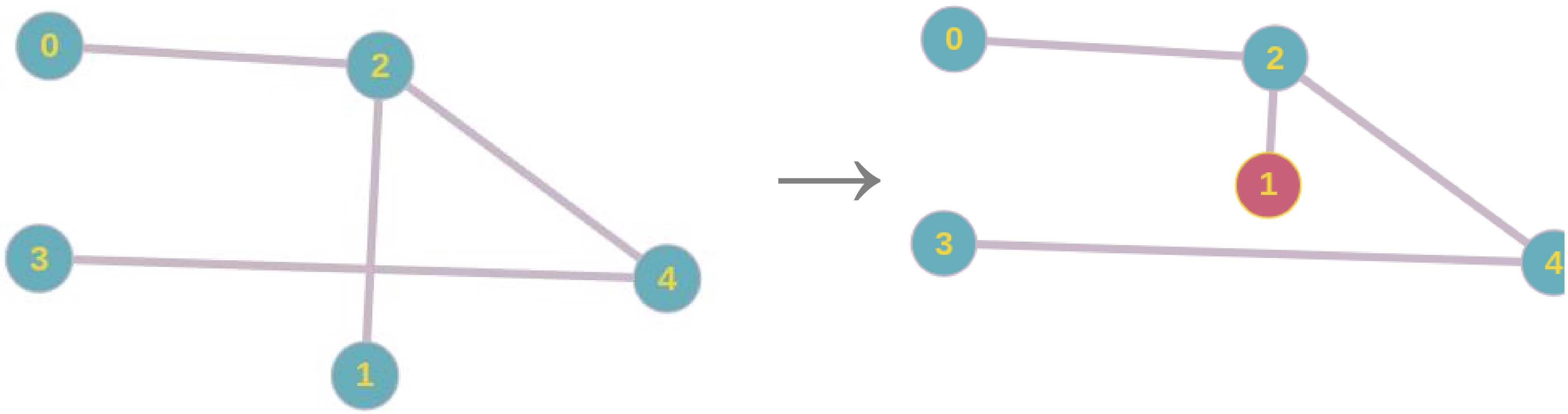
- Attempts to find global optimum
- Energy function to determine fitness of solutions
- "Annealing" Principle:
    - Initial high "Temperature" value, decreasing with time
    - Alters solution (switches to neighbouring solution) if:
        - Neighbouring solution has a lower energy or
        - Neighbouring solution has a higher energy and the temperature is high



Implementation

- Inspired by d3-labeler
- Definition of "neighbouring solution" per iteration:
    - Movement of all nodes?
        - Takes longer, but may lead to faster convergence
    - Movement of one node?
        - May need more iterations for good result
        - Better for demonstration purposes

Implementation



Accept if:

Energy(new) < Energy(old)

or

Math.random() >= Math.exp(-delta_energy / currentTemperature)

Energy Function (Example)

```
function energy(graph) {
  let energy = 0;
  for (let edge of graph.edges) {
    for (let otherEdge of graph.edges) {
      if (intersection(edge, otherEdge))
        energy += weight_line_intersection;
    }
```

Energy Function (Example)

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      if (intersection(edge, otherEdge))
        energy += weight_line_intersection;
    }
    for (let node of graph.nodes) {
      if (intersection(edge, node))
        energy += weight_line_node_intersection;
    }
  }
```

Energy Function (Example)

```
function energy(graph) {
  let energy = 0;
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      if (intersection(edge, otherEdge))
        energy += weight_line_intersection;
    }
    for (let node of graph.nodes) {
      if (intersection(edge, node))
        energy += weight_line_node_intersection;
    }
  }
  for (let node of graph.nodes) {
    for (let otherNode of graph.nodes) {
      if (intersection(node, otherNode))
        energy += weight_node_intersection;
    }
  }
  return energy;
}
```

Demo

Start Annealing | Small Graph ▼

Current Energy: N/A

☑ **node intersection** Prevents nodes from overlapping.

weight 30.0

☑ **edge intersection** Prevents edges from overlapping.

weight 30.0

☐ **edge-node intersection** Prevents edges from passing through nodes.

weight 10.0

☐ **edge length** Penalty for length of edges.

Tolerated Length: 20

Weight 0.1

Evaluation

Advantages:

- Can stop after any amount of iterations
- Custom criteria are easy to implement and extend

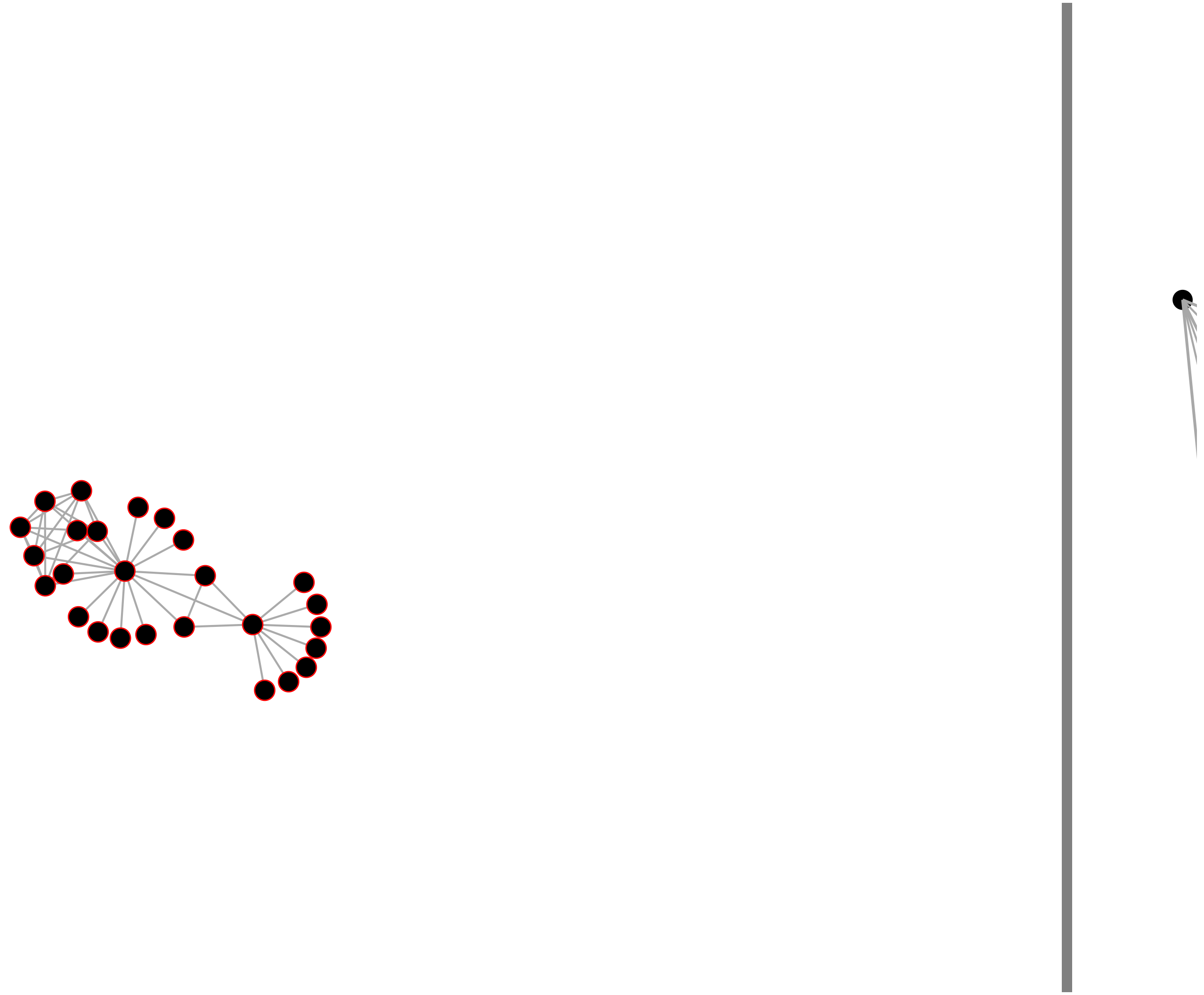
Disadvantages:

- High runtime ($O(n^4)$)
- Less applicable for larger graphs

Force-Layout Graph vs Simulated Annealing

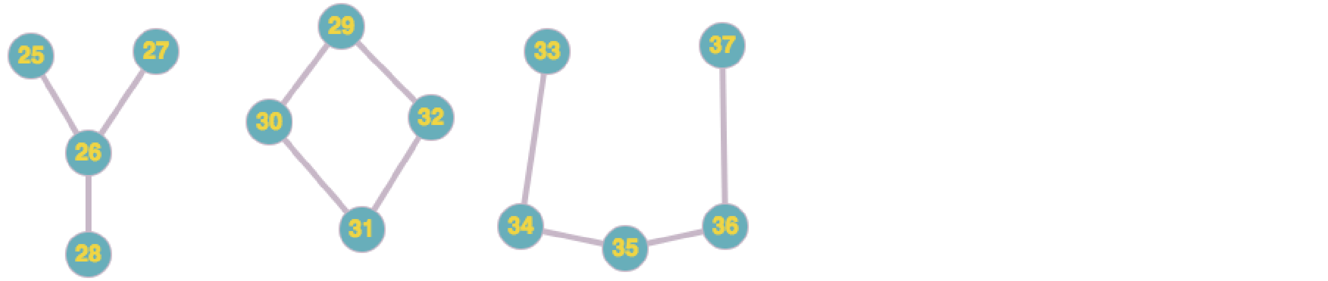
Demo

Refresh Annealing | Medium Graph ▼ | Graph Energy: 1022



Conclusion

- Simulated Annealing easily customizable, but less applicable for large graphs
- Force layout provides widely applicable, reasonably fast and interactive solution



References

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